**Random Vectors**

**Generalization** of the concept in **ch. 4** **to any # of random variables**

* Vector notation
* Matrix notation

**Probability model with  random variables**



* **Multivariate Joint CDF**

The joint CDF of  is



* It provides a complete probability model regardless of discrete, continuous, or mixed.
* **Multivariate Joint PMF**

The joint PMF of discrete random variables  is



* **Multivariate Joint PDF**



If  are **discrete random** variables with joint PMF 

* 
* 

If  are **continuous random variables** with joint PDF 

1. 
2. 
3. 

The probability of an event  expressed in terms of the random variables  is

**Discrete:**



**Continuous:**



**Ex5.3** The random variables  have the joint PDF



Let A denote the event that. Find.



**Quiz 5.1**: The random variables  have the joint PDF



Let C denote the event that. Find 



**5.2 Vector notation**

* When an experiment produces two or more random variables, vector and matrix notation provide a **concise representation** of probability model and their properties.

In general, the column vectors are used



where  indicates transpose of vector or matrix.

**Random vector probability functions**

1. **CDF** (Cumulative Distribution Function) of a random vector  is



b) **PMF** (Probability Mass Function) of a discrete random vector  is



c) **PDF** (Probability Density Function) of a continuous random vector  is



**Probability function of a pair of Random Vectors**

For random vectors  with  components and  with components:

1. The joint CDF of  and  is



1. The joint PMF of discrete random vectors  and  is



1. The joint PDF of discrete random vectors  and  is



Ex. The fair of random vectors  and  is the same as



**Ex 5.4)** Random vector  has PDF



where



What is CDF of ?



Because  has three components, we assume that is a 3-dimensional vector.



**Quiz 5.2)** Discrete random vectors  and  are related by  . Find the joint PMF  if  has joint PMF







**5.3Marginal probability function**

For a joint PMF  of discrete random variables  some marginal PMFs are



For a joint PDF  of discrete random variables  some marginal PDFs are



**Ex 5.5)** The random variables  have the joint PDF



Find the marginal PDFs  and 





Quiz 5.3) The random vector  has PDF



Find the marginal PDFs



i)



ii)



iii)



iv)



iv) Let’s try with **different order of integration** to find  again



v)



vi)



**5.4 Independence of random variables and random vectors**

Independent random variables

Random variables  are independent if for all,

Discrete: 

Continuous: 

**Ex** Revisiting the previous example and see whether it is independent or not.

The random vector  has PDF









= , , 

 **not independent**